Australasian Journal of
Educational Technology

2010, 26(1), 85-104

Building mathematical knowledge in an authentic mobile phone environment

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Although many researchers have examined knowledge building in traditional settings and distance learning, few have examined middle school students' building of mathematical knowledge using mobile phones. The present study uses two well-known models of knowledge building to carry out the examination: the interactive analysis model of knowledge building phases developed by Gunawardena, Lowe and Anderson (1997) and the six themes model of knowledge building characteristics developed by Scadamalia and Bereiter (2006). The findings show that the middle school students participating in this research went through all the knowledge building phases suggested by Gunawardena, Lowe and Anderson (1997). They further experienced other knowledge building phases that fit the authentic context in which they learned. Participants advanced their knowledge of ideas as a community, collaborating to carry out authentic activities using mobile phones. They demonstrated constructive and critical use of information in general and of authoritative information in particular. Participants worked as mathematicians, especially during the second part of the experiment, when they suggested real world phenomena to explore using the mobile phone. My conclusion suggests learning mathematics by carrying out authentic activities using mobile phones, to encourage and enrich the mathematics knowledge building of students in K-12.

Introduction

Silander and Rytkönen (2005) found that mobile devices bring a new dimension to learning and education because they allow learning to occur in authentic contexts and extend to real environments. These authentic contexts and real environments, together with the mobile devices, enable student learning activities characterised by communication (afforded by the mobile devices), collaborative knowledge building (encouraged because of the complexity of authentic contexts), observation (facilitated by the mobile devices, especially by their cameras), and innovation (felt due to using new devices and new contexts). Thus, using mobile devices students are able to construct useful knowledge in real situations. This turns the mobile devices into powerful tools in the hands of students, but what characteristics of knowledge building do these powerful tools bring to students' learning? This is what this research attempts to answer.

Background

Mobile mathematics education

Despite the ubiquity of mobile phones in every aspect of our daily lives, the use of these devices in education is still new (Chen and Kinshuk, 2005) and in its infancy

(Rismark et al., 2007). This is true for the use of mobile phones in education in general and especially for their use in teaching mathematics. At the same time, mobile and wireless devices, especially mobile phones, have recently become increasingly common among young students. This provides new possibilities, opportunities, and challenges for education (Cobcroft et al., 2006), and especially for mathematics education. Recent studies that examined the use of mobile phones in mathematics learning among pre-service teachers (Botzer & Yerushalmy, 2007; Genossar, Botzer, & Yerushalmy, 2008) suggest that we are at the beginning of a new era for mobile phone integration in the mathematics classroom.

Genossar, Botzer and Yerushalmy (2008) studied the learning processes and experiences taking place within a mobile phone learning environment and examined how socio-cultural and situated learning aspects are reflected in these processes and experiences. They found that the contribution of the mobile phone environment "lies not only in making dynamic mathematical applications more available, but also in supporting the execution of tasks that are closer to the students' experiences and more relevant to them, which has the potential to enhance experiential learning." The authors concluded that the participants' learning experiences contributed to their personal learning, which in turn motivated this learning. Genossar, Botzer, and Yerushalmy (2008) experimented with pre-service teachers, and the participants worked individually, whereas the study reported here is concerned with middle school students, and the participants worked at the beginning in groups of 4-6 students and later in a whole-classroom setting.

White (2004, 2006) explored the use of wireless handheld computers to support middle school students' collaborative learning of algebraic functions in an applied context of cryptography. The wireless computer network used by the students blended multiple linked representations of mathematical functions with role based student group work, to facilitate the learning of algebraic functions and the solving of complex mathematics problems in small groups. White (2006) reported that working in a network of handheld computers, students "simultaneously negotiated shared utterances through a discursive network and shared objects through a device network" (p. 380). The network added to the participatory opportunities of classroom collaboration, and lower performing students, working in a network, enhanced their achievement. The collective artifacts were not only network based but also mathematically rich. White (2004, 2006) experimented with learning in a network inside the classroom, and the students were assigned specific roles at the beginning of the activity and could alternate their roles later, whereas this study is concerned with mathematics learning both outside and inside the classroom, and the participants could decide from the beginning what roles to assume.

Roschelle and colleagues conducted several experiments using mobile devices in the mathematics classroom. For example, Tatar, Roschelle, Vahey & Penuel (2003) examined the use of mobile devices in mathematics and science learning by implementing several activities that became possible owing to the availability of mobile devices, including: (a) distribution: sending the same document to all students, (b) differentiation: sending different parametric definitions to each student in a systematic way, (c) contribution: forwarding a function or mathematical data constructed by one student to a friend or teacher, (d) harvesting: following the collaborative work of several students, constructing a set of functions or data that are related to each other but different; and (e) aggregation: combining functions or data that are in some way related and presenting it usually in public (anonymously or not). The study found that mobile learning promises access to applications that support learning anywhere, anytime, and that this type of learning supports both adults at the workplace and students in classroom learning. Roschelle, Patton & Tatar (2007) found that the use of mobile devices in the mathematics classroom made the class more (a) student centred, (b) assessment centred, (c) knowledge centred, and (d) community centred.

In the last decade, several researchers have been following the integration of applets (which are common on the Internet) in the mathematics classroom. Applets help students study mathematics using a constructivist approach (e.g., Pesonen, 2003). As an extension to the use of these tools within a web environment, recently new mathematical applications have become available for mobile devices, and most recently for mobile phones. These applications are called *midlets*. *Wikipedia* defines midlets as Java programs for embedded devices, generally games and applications that run on a mobile phone (*Wikipedia*, 2008).

As applets replaced courseware and dedicated tools in computers, midlets replaced applets in mobile phones. But the unique learning environment of mobile phones includes, in addition to the midlets, such features as the ability to take pictures, record video and audio, transfer information, use voice and text communication, forward screen content to learning mates, and send SMS (Short Message Service: a communication protocol that enables the interchange of short text messages between mobile telephone devices) and MMS (Multimedia Messaging Service: a mobile phone standard for sending messages that include multimedia objects such as images, audio, video, and rich text) messages. It seems that in addition to the availability, mobility, dynamics, and accessibility of the mobile phones, these features can make a difference in the way mathematics students build their knowledge.

Regarding research which examined the students' perceptions of learning with mobile phones and applets, Daher (2009) examined the perceptions of middle school students about the use of mobile phone midlets and web applets in learning mathematics and how they differentiate between the two tools. Daher (2009) reported that the students were aware of the following aspects of each one of the tools: its availability, its portability, its collaboration aspect, its communication aspect, the size of its interface, and its usability. The students used these aspects to describe their experience in using the tools to learn mathematics, to differentiate between them, and to decide which tool they would use in their future learning and how they would use each tool. A higher proportion of students preferred the mobile phone as a learning tool because of its portability and communicability.

Regarding research which examined mathematics learning by middle school students who used the mobile phone, Baya'a and Daher (2010) examined the conditions that influenced middle school students' learning of mathematics, when using the mobile phone, and, at the same time, the consequences of such learning. They found that what affected the students' learning in the mobile phone environment were the characteristics and technologies of the mobile phone, the requirements and topics of the mathematical activities, the learning setting (inside or outside the classroom), the intention of the researchers who participated in the teaching processes, and the involvement of the school principal and the coordinating teacher. The consequences of the mathematics learning in the mobile phone environment were: the students took control of their learning, they connected mathematics with real life phenomena, they developed a new approach to mathematics where they looked at it as an applied science, and the students worked as mathematicians.

Authentic learning

Eble (1988) suggested that students understand better and apply better study materials when they are engaged in real world issues and situations. Quitadamo and Brown (2001) showed that authentic situations and scenarios stimulate student learning, thereby creating greater motivation and excitement for it. They added that representing and simulating real world problems and contexts provides an important context for the students' thinking. Silander et al. (2004) showed that mobile devices extend the learning environment in which the students work and integrate it with real-life situations, where learning can occur in an authentic context. According to Ting (2007, p. 718), "mobile learning can guide a learner to an authentic learning context and incorporate the field objects with closely related information in the handheld device to initiate the process of knowledge acquisition."

Herrington et al. (2008) identified nine characteristics of authentic learning:

- *authentic contexts* that reflect the way the knowledge will be used in real life
- *authentic activities* that are complex, ill-defined problems and investigations
- *access to expert performances* enabling modeling of processes
- *multiple roles and perspectives* providing alternative solution pathways
- *collaboration* allowing for the social construction of knowledge
- *opportunities for reflection* involving metacognition
- *opportunities for articulation* to enable tacit knowledge to be made explicit
- *coaching and scaffolding* by the teacher at critical times
- *authentic assessment* that reflect the way knowledge is assessed in real life

Most of the above characteristics characterise the learning of the students participating in the research, when they solved real world problems with mobile phones.

The authentic learning in our case involved mathematical modeling, where mathematical modeling connects mathematics and reality, and it involves making representations, in mathematical terms, of the problem being studied, using numerical expressions or formulas, diagrams, graphs or geometric representations, algebraic equations, tables, etc. (Ferreira & Jacobini, 2008).

Knowledge building

Scardamalia (2005, p. 350) described knowledge building as an "activity focused on the generation of new knowledge and the continual improvement of ideas," and added that it "requires that ideas be revisited, revised, linked to other ideas, raised to higher status, reframed in light of new findings, and evolved into new forms." She described knowledge builders as producing "ideas that have a life beyond their own minds, beyond personal notebooks, and beyond short-lived discussions." (Scardamalia, 2005).

Knowledge building characteristics

To provide an educational space in which students can advance their knowledge building, overcoming the obstacles mentioned above that characterise learning in traditional schools, Scardamalia (2002) identified 12 characteristics necessary for knowledge building spaces: (a) real ideas, authentic problems, (b) improvable ideas, (c) idea diversity, (d) rising above, (e) epistemic agency, (f) community knowledge, collective responsibility, (g) democratisation of knowledge, (h) symmetric knowledge advancement, (i) pervasive knowledge building, (j) constructive use of authoritative sources, (k) knowledge building discourse, and (l) embedded and transformative assessment.

Scardamalia and Bereiter (2006) noted six themes that should characterise student learning in a space that encourages knowledge building: (a) knowledge advancement as a community rather than individual achievement (this theme assumes that knowledge doesn't just accumulates, but advances. When it does so it is through the effort of the community and not just the individual), (b) knowledge advancement as idea improvement rather than as progress toward true or warranted belief (advancement of knowledge should take care of improvement of ideas and not of a required or final state of knowledge), (c) knowledge of, in contrast to knowledge about (this theme emphasises the knowledge of doing something and not just knowing about something), (d) discourse as collaborative problem solving rather than as argumentation (this is the discourse committed, through problem solving, to progress, to seek common understanding, and to expand the base of accepted facts) (e) constructive use of authoritative information (authoritative information should be used only when needed and to advance the learner's knowledge), and (f) understanding as an emergent phenomenon (ideas can interact with one another to produce new and more complex ideas).

Knowledge building phases

There is more than one model or framework for evaluating student knowledge building phases (Veerman & Veldhuis-Diermanse, 2001; Fahy et al., 2001; Gunawardena, Lowe & Anderson, 1997). Veerman and Veldhuis-Diermanse (2001) described a five-phase knowledge building model in which the learners acquire the following: phase 1 – new facts, phase 2 – new experiences, phase 3 – new theory, phase 4 – explicitation, and phase 5 – evaluation.

Gunawardena, Lowe, and Anderson (1997) also suggested a five-phase model for analysing online conferencing. The phases of this model, referring to the learning that takes place in the online conferencing, are described below (following Kanuka & Kreber, 1999):

Phase I: Sharing/comparing of information. In everyday transactions this can take the form of ordinary observations, statements of problems, or questions. This phase may include an observation, opinion, agreement, corroborating example, clarification, and/or identification of a problem.

Phase II: Discovery and exploration of dissonance or inconsistency among ideas, concepts, or statements advanced by different participants. This is defined as an inconsistency between a new observation and the learner's existing framework of knowledge and thinking skills. Operations that may take place within this phase can include the identification of differences in the understanding of terms, concepts, schemas, and/or questions needed to clarify the extent of disagreement.

Phase III: Negotiation of meaning and/or co-building of knowledge. This phase includes negotiation or clarification of the meaning of terms, identification of areas of agreement, and a proposal of a compromise or co-building.

Phase IV: Testing and modification of the proposed synthesis or co-building. Events that occur in this phase include testing against an existing cognitive schema, personal experience, formal data experimentation, or contradictory information from the literature.

Phase V: Phrasing of agreement, statement(s), and the application of the newly constructed meaning. This phase encompasses summarising agreement(s) and metacognitive statements that illustrate new knowledge building and application.

Research rationale and objective

Although mobile phones are tools that students enjoy using, few projects or experiments have been carried out on their use in the mathematics classroom (Baya'a & Daher, 2009). This explains why little research has been performed on students using mobile phones to build mathematical knowledge, and even less in authentic contexts, especially research focusing on middle school students involved in modeling mathematically real world phenomena outside the classroom. The studies described above were performed about students' mathematical learning using mobile devices in various settings (White, 2004, 2006; Botzer & Yerushalmy, 2007; Genossar, Botzer, & Yerushalmy, 2008; Daher, 2009; Baya'a & Daher, 2010), but there has been little research on middle school students' building of mathematical knowledge using mobile devices outside the classroom. The present study attempted to examine such an issue when middle school students collaborate to carry out activities involving real world phenomena. This study suggests how the students can work outside the classroom to learn about real world phenomena mathematically, model the phenomena using mobile phones, discuss the functions' graphs and the algebraic rules that they obtain, and reflect about their work in a whole class setting. This can open new opportunities for mathematics learning.

Research questions

The study posed the following research questions:

- What are the knowledge building collaborative phases of learning mathematics by solving authentic problems in a mobile phone environment?
- What characteristics of knowledge building does a mobile phone environment offer to the learning of mathematics in an authentic context?

Method

Research setting and sample

The experiment took place in a middle school in Baka in Israel from mid-January to mid-April in the academic year 2008-2009. It was led by three pre-service teachers majoring in mathematics and computers in Al-Qasemi Academic College of Education. The preservice teachers carried out a project in teaching mathematics using mobile phones. The project was the preservice teachers' main task in a mathematics didactics course which emphasised the role of technology in mathematics education. The pre-service teachers selected thirty 8th grade students (age range 14-15 years) to participate in the project. The selection was based on the interest of the students and ownership of an appropriate mobile phone (not all the selected students had an appropriate mobile phone, and some of them asked their parents to buy for them a Java-enabled phone). The students had no previous knowledge about the topic of functions. Part of the learning was performed by means of outdoor activities that involved exploring the mathematics of real life phenomena. The students took advantage of the various characteristics of mobile phones in their explorations. The other part of learning was performed in the classroom, where the students discussed the graphic and algebraic results that they obtained and reflected on their work outside the classroom.

The students worked in groups of 4-6. They were required to find mathematical relations in real world phenomena and worked on these relations out of class. The students decided themselves which roles they would play (measuring, observing, writing down the observations, assigning points in midlets, taking pictures, etc.), and made decisions about altering these roles when necessary. The students also discussed the results that they obtained, referring to the graphs and algebraic rules that fit the real world phenomena. This discussion was carried out in the classroom.

Initially, the students performed the activities suggested by the pre-service teachers. Later in the experiment, after the students had carried out eight real world activities, they started to develop activities themselves by suggesting real world activities they judged to be executable with mobile phones. The students usually started from a specific suggestion and proceeded to develop it until they considered it to be worth performing.

The mathematical software

The middle school students worked with mobile phone software (midlets) that support the learning of algebra and geometry. The midlets can be downloaded from *Math4Mobile* site which belongs to the Institute for Alternatives in Education that operates within the Faculty of Education at the University of Haifa (Yerushalmy & Weizman, 2007). To perform the activities assigned to them, the students used the algebraic midlets and various tools and technologies embedded in their mobile phones. The students used mostly the *Fit2Go* midlet, which enables users to draw specified points and fit a linear or a quadratic function to them. When a student needs the midlet to fit a linear or a quadratic function to some points, the midlet provides the graph and algebraic rule of the function if such a function exists; if not, it displays the graph and algebraic rule of a function that passes through some of the points drawn. Figure 1 shows a quadratic function that the *Fit2Go* midlet provided for 5 drawn points, where the function passes through 3 of the 5 points.

Activities

Initially, the pre-service teachers asked students to carry out the following real world activities:

1. Find the relation between the weight and the height of group members. The students weighed each other and measured each other's height, then assigned points using the *Fit2Go* midlet, with each point representing the measurements taken for one student (weight for x and height for y).
2. Find the relation between the number of sons and the number of daughters in each family. Students indicated the number of sons and daughters in their families and in their relatives' families, then assigned points in *Fit2Go*, with each point

representing the number of one family (number of boys for x and number of girls for y).
3. Find the relation between the circumference of the trunk of a tree and the circumference of one of its branches.
4. Find the relation between the radius of a car's tires and its height.
5. Find the relation between the time it takes to fill a container and the height of the water in it. The students worked with different container shapes, one at a time.
6. Find the relation between a person's height and speed.
7. Find the relation between the time that passes from the moment a ball has been thrown and the distance that the ball travels.
8. Find the relation between a person's weight and the number of fingers on his/her hand. Students were required to study this relation in order to arrive at the constant function, i.e., to recognise that some real world relations are constant.

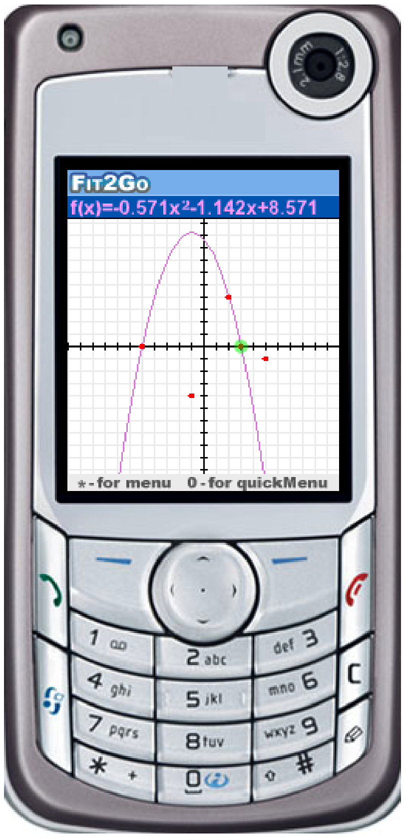

Figure 1: Quadratic function that fits 3 of 5 drawn points

After the students performed the activities suggested by the pre-service teachers, they were asked to make their own suggestions about authentic activities that they could carry out with mobile phones. The activities suggested by the students were:

1. Find the relation between the temperature of the water in a container and the time required for an ice cube to melt in that water.

2. Find the relation between the circumference of a rock and its height.
3. Find the relation between the length of a leaf and its width.
4. Find the relation between the height of a person and the length of his/her step.
5. Find the relation between the time that elapses from planting a plant and the plant's height.
6. Find the relation between the time that elapses from lighting a candle and its height at that moment.

Data collection tools

The pre-service teachers used various means to collect data about the participants' learning of mathematics with mobile phones: observations, filming videos, writing in a blog, and interviews. The data collected by the pre-service teachers was used to analyse the students' building of mathematical knowledge in solving authentic problems using the mobile phone. In addition, the pre-service teachers kept diaries to reflect the flow of the experiment.

Data analysis

Coding the knowledge building phases
The interactive analysis model developed by Gunawardena, Lowe, and Anderson (1997) was used to determine the knowledge building phases.

Coding the knowledge building characteristics
The six themes described by Scadamalia and Bereiter (2006) were used to determine the knowledge building characteristics. The coding was carried out on a group basis because the students performed the activities in groups.

Relevance of the data analysis to the setting and theme of the study

The objective of the study was to find (a) the knowledge building collaborative phases in the process of middle school students solving authentic problems in a mobile phone environment, and (b) the characteristics of students' knowledge building in an authentic context that uses mobile phones.

Gunawardena, Lowe, and Anderson (1997) suggested an interaction analysis model for examining the building of knowledge in computer conferencing. Many researchers used this model to analyse students' knowledge building; for example Kanuka and Kreber (1999) used the model to analyse the phases of knowledge building in the virtual classroom, and Tan, Chai, and Hong (2008) used the model to analyse small group knowledge building effort among teachers. Therefore, the model appears to be appropriate for analysing the phases of middle school students' knowledge building when using mobile phones.

The work of Scardamalia and Bereiter is acknowledged to be seminal in the field of knowledge building. Gramlinger and Czerwionka (2004) used the twelve knowledge building themes of Scardamalia (2002) to develop and evaluate course sequences for school, and Law and Wong (2003) used 10 of the 12 knowledge building themes to evaluate the level of performance of groups of students. I used Scardamalia's and Bereiter's (2006) more recent knowledge building themes to analyse and evaluate the characteristics of students' knowledge building of mathematics in authentic contexts when using mobile phones as their technological tools (which include Scardamalia's original themes). For example, Scardamalia's (2002) community knowledge, collective responsibility, and democratisation of knowledge can be considered under knowledge advancement as a community rather than an individual achievement according to Scardamalia and Bereiter (2006).

Findings

Knowledge building phases in learning mathematics outdoors by solving authentic activities

The phases are described using the interactive analysis model developed by Gunawardena, Lowe, and Anderson (1997).

Solving authentic mathematical problems with mobile phones, the middle school students went through the following phases to build their mathematical knowledge about modeling real world phenomena mathematically.

Phase I - Planning:

- Planning the activity setting: which trees to measure, which rocks to measure, etc.
- Planning who will carry out each part of the activity: who will measure the trees, who will write down the measurements on paper, who will assign the points in the midlet, who will record the activity, etc.

Phase II - Carrying out the physical part of the activity:

- Preparation: lighting the candle, finding out whether the stairs are clean so that one can climb them easily, arranging with the nurse the time of taking the measures of the group members' weight and height, etc.
- Measurements: measuring the circumference of the tree trunk, measuring the time it takes to climb the stairs, telling the number of sons and the number of daughters in a family, etc.
- Photographing and filming the activity.
- Writing the measurement outcomes on paper.

Phase III - Modeling the real life phenomenon:

- Entering the points resulting from the measurement in the midlet.
- Fitting a function to the assigned points.

Phase IV - Sharing of experience and results:

- Describing the experience of performing the activity.
- Showing pictures or videos of the activity.
- Describing difficulties encountered in carrying out the activity: describing difficulties in lighting a candle outdoors, in measuring the circumference of large rocks, in making a person keep the length of his/her steps when walking, etc.
- Describing the measurement outcomes: describing the heights of group members and the lengths of their step, etc.
- Describing properties of the resulting function, graphically and algebraically: a linear function, an increasing function, the function is positive when …, etc.
- Comparing the resulting functions, graphically or algebraically: determining whether all the functions that represent the relation between the weight and height of group members increase from left to right, whether they increase with the same

rate of change, determining whether function parameters should accept some values and not others (for example, determining whether the parameter *a* in the functions 'f(x) = ax + b' or function 'f(x) = ax^2 + bx + c' should be positive or negative), etc.

Phase V - Inconsistency in results, ideas, or concepts:

- Talking about differences in the properties of the functions describing the real world phenomenon or about disagreement about a mathematical idea or concept.

Examples:
When discussing the linearity of the function that represents the relation between the weight and the height of members of one of the groups, one student said that a straight line is sure to pass through any three points. Another student disagreed, saying a straight line never passes through three points, and that what passes through three points is a triangle.

When describing the function that represents the relation between a person's weight and the number of fingers on his/her hand, some students said that what resulted was not a function because no x appears in the algebraic rule, while others recognised the result as the constant function.

Phase VI - Negotiation of meaning/co-building of knowledge:

- Clarifying the meaning of terms, ideas, or concepts in areas of agreement and disagreement.
- Challenging and influencing each other's reasoning.
- Discussing the source of differences or disagreement.
- Explaining the results obtained.

Examples:
Discussing the previous first issue, students drew triplets of points and examined possible straight lines that could pass through them. The students said that one source of the problem is that they immediately imagine a triangle when a triplet of points is mentioned.

Discussing the previous second issue, students discussed when a graph is a graph of a function and when an algebraic rule is a rule of a function. This settled the issue for the students who did not recognise the graph as that of a function.

Phase VII - Agreement:

- Agreeing on the possible results of an experiment.
- Agreeing on the meaning of the results obtained and on the mathematical meaning of the ideas and concepts associated with the real life activity.

Example:
The students agreed on the non-linearity of the function that represents the relation between the weight and height of a person. They explained that the non-linearity resulted from some bodies having more or less weight than it is ideal. Some students said that if the weight matches the height the body looks beautiful, whereas other students suggested a formula for calculating the ideal weight for males and females. The students further indicated that the mismatch between weight and height can be a function of various parameters: how much one eats, how much one exercises, how the body processes food, etc.

To account for the students' next activity (suggesting real world activities to carry out using the mobile phone), we must add the following initial phase to the above set of phases:

Phase 0 - Developing mathematical contexts to explore
- Suggesting a real world phenomenon to explore with the mobile phone.
- Developing the suggestion.

Characteristics of knowledge building in the process of learning mathematics

The characteristics are described using the framework suggested by Scardamalia and Bereiter (2006).

- *Advancement of knowledge as a community rather than an individual achievement.*
  The students advanced their knowledge as part of a community, which included the pre-service teachers who led the experiment. The students expressed their enthusiasm because they carried out all the phases of the activity in equal collaboration with their teachers.

  Before the experiment, the participants had not performed authentic activities which included mathematical modeling. They had no prior knowledge of mathematical functions (the subject is not included in their curriculum), and were not accustomed to justifying logically the results they obtained. By the end of the experiment the participants as a group advanced their knowledge about mathematical modeling of authentic activities, about functions and their properties, the planning and execution of authentic activities, the design of authentic mathematical activities, and the analysis of their mathematical actions.

- *Advancement of knowledge as improvement of ideas rather than progress toward true or warranted belief.*
  Progress toward a warranted belief of the students occurred during the physical phase of the activity and during the graphing phase, in working with the midlet. These phases resulted in beliefs about the true properties of the function that represented the real world relation. The progress as idea improvement occurred during all the learning phases. According to the pre-service teachers' testimony, the students became expert in all the phases of the experiment, from planning to discussion and reflection. One of the pre-service teachers noted:

  > The students learn new things in each activity. This makes them advance their knowledge and improve their performance.

Another pre-service teacher wrote:

> The students advance their knowledge in different aspects, specifically, they now carry out the activity in a scientific way, for example, they threw the ball hard to make it go in a straight line. They now give opinions that are based on mathematical justification. They work in an environment full of challenge, whether they execute the physical part of the activity, graph the obtained relation, or justify the algebraic rule obtained."

- *"Knowledge of" as opposed to "knowledge about".*
  The students did not learn about functions directly from the teacher, and therefore they had no knowledge of the concept of a function before the research experiment. The experiment helped students become familiar with functions by several means:

(a) solving real world problems and identifying which real world phenomena could be translated mathematically using the mobile phone midlets, (b) modeling the real world phenomena with the midlets, and (c) discussing the properties of the obtained function's graph and algebraic rule.

In other words, the students participating in this experiment did not *obtain* knowledge about functions but *constructed* knowledge of functions and of their properties. The real world context enabled students to perform physical actions, like measuring lengths, weighing weights, planting, etc. At the same time, modeling real world problems with mobile phone midlets helped them view different mathematical representations (algebraic and graphic) of the same real world phenomenon. This introduced the students to three representations of functions at the same time, indeed four representations if we consider the real world context to be another representation of functions. It also helped students construct their knowledge of the various aspects of functions in an authentic context.

- *Discourse as collaborative problem solving rather than argumentation.*
  Activities were carried out collaboratively. One of the pre-service teachers wrote in her diary:

> If one of them found difficulty in understanding any step, the other students helped him. If the whole group confronted a difficulty they came to us, and we discussed the issue.

Another pre-service teacher wrote:

> Although some actions were performed by the students individually, like weighing or measuring the height of the students, the overall activity was performed in a group spirit, because when one student measured another's height, the third one registered the measurement, and the rest of the members of the group watched, sometimes suggesting corrections if the height wasn't measured or recorded correctly.

When discussing the experiments, the students presented supporting evidence about the results they obtained and described the nuances of the activities. They also investigated the mathematical relations that underlie the results, stating for example that a positive *a* (the slope) implies an increasing function when the function is linear. The students explained this by stating that "a positive *a* increases the value that we get from a function when we increase the input to the function." Some of the students were not satisfied with this explanation and wanted to discuss the reason why increasing the input increases the output, so they examined cases of positive and negative values of *a*. They also depended on the multiplication rules to explain why a positive *a* increases and a negative *a* decreases the output when increasing the input.

Initially, the pre-service teachers asked students to justify everything they said. The students found this difficult at first, but eventually they started to ask each other to justify their statements and to base their claims on facts or on known mathematical relations.

- *Constructive use of authoritative information.*
  The pre-service teachers did not volunteer to provide students with information. Only when the students consulted them on how to proceed, did the pre-service teachers furnish any information. This information was never a definite answer but a suggestion of one of the possibilities that may be pursued.

  One of the pre-service teachers described how the students used the information provided by others: "The students watched each other carry out the activity. They noted that the measurement wasn't precise and required that they be taken again." Another pre-service teacher wrote:

  > The student said that the findings were unreasonable and suggested to bring again candles, light them and measure how long it takes different candles to burn down.

  In this way, the students considered the information critically, questioned it, and tried to obtain more accurate information.

- *Understanding as an emergent phenomenon.*
  The pre-service teachers, as mentioned above, did not volunteer mathematical or any other type of information to the students, for example, concerning the planning of an activity. They interfered only when the students asked them to do so. In these cases the teachers tried not to lead the experiment but to offer suggestions on how to proceed. Thus, the students arrived at understanding on their own. This understanding emerged in the course of the various phases of activity - not only during the physical, graphing, or negotiating parts but throughout the entire activity. Students' understanding emerged as part of the process of discovering the relations between the various phases of the activity and the various representations of the mathematical phenomenon, idea, or concept.

Discussion

Knowledge building phases

The knowledge building phases identified in this study differ from those introduced by Gunawardena, Lowe and Anderson (1997), which can be explained by the difference in the context of the experiment. The interaction analysis model suggested by Gunawardena, Lowe, and Anderson analyses the interactions occurring in online conferencing, whereas the present model analyses knowledge building in an authentic context. This adds two phases to the model: "performing the physical part of the activity" and "modeling the real life phenomena." Another difference is that the students were required to plan the activities, which adds a planning phase to the model. The rest of the present model is similar to that of Gunawardena, Lowe and Anderson, although the phase "testing and modification of proposed synthesis or co-building" was incorporated in "negotiation of meaning/co-building of knowledge" because the testing and modification of the proposed synthesis was considered to be part of the last synthesis performed by the co-builders of knowledge.

The findings show that middle school students who participated in the study went through all the phases of knowledge building suggested by Gunawardena, Lowe and Anderson, which underscores the effectiveness of learning mathematics in an authentic context using mobile phones.

Knowledge building characteristics

Community knowledge advancement
Scardamalia and Bereiter (2006) noted that "knowledge building pedagogy is based on the premise that authentic creative knowledge work can take place in school classrooms." This is what happened in this experiment with the building of mathematical knowledge. Eighth grade students as a group were able to advance their knowledge of mathematical relations, to carry out authentic activities and model mathematically real world phenomena.

Scardamalia and Bereiter (1994) argued that

> ... the classroom needs to foster transformational thought, on the part of both students and teachers, and that the best way to do this is to replace classroom-bred discourse patterns with those having more immediate and natural extensions to the real world, patterns whereby ideas are conceived, responded to, reframed, and set in historical context.

This is what the pre-service teachers and the middle school students did when using mobile phones as technological means with which the students solved mathematical problems and modeled real world phenomena represented by the problems.

Knowledge advancement as idea improvement rather than as progress toward true or warranted belief
Students improved their ideas in several areas: mathematical properties and relations, planning activities, executing them, and analysing their actions. Wang (2000) introduced the use of technology to construct a learning environment for advancement through three levels of knowledge: *know what, know how*, and *know why*. Using mobile phones to learn mathematics in an authentic context produces an environment for advancing all three levels of knowledge:

- *Know what:* the students improved their ideas about what phenomena could be represented in a linear relation and which ones using the *Fit2Go* midlet.
- *Know how:* the students improved their ideas about how to plan authentic activities, how to carry them out, and how to graph activities.
- *Know why:* the students improved their ideas about why the function representing real life phenomena is linear, increases from left to right, has a positive slope, etc.

In sum, using a mobile phone to carry out authentic mathematical activities provided a rich environment for idea improvement.

Knowledge "of" as opposed to knowledge "about"
Oblinger (2007) noted that authentic learning "allows students to use the practices of professionals to gain experience, understanding and motivation." In this experiment, authentic learning performed with the help of mobile phones enabled students to investigate mathematical activity through actions and to model the mathematical phenomena by representing the activity with the aid of mobile phones. The two processes, together with the analysis of the outcomes of the activities, including the graphs and algebraic rules, enabled the students to build knowledge of real life problems that could be subject to mathematical investigation; to build knowledge of mathematical modeling; and to act as mathematicians who build knowledge of mathematics and of mathematical work.

Discourse as collaborative problem solving rather than as argumentation

The students performed the physical part of every activity that involved problem solving collaboratively, and collectively graphed the obtained relation in the problem. In so doing, they were prepared for the upcoming discussion of results, which is by nature a collaborative activity. The fact that the first phases of the activity had been carried out collaboratively made it easier for the students to have a common ground when discussing, analysing and justifying their findings.

Discoursing and reflecting on their findings, the participants reflected not only on their findings but on their discussions as well. They did not accept unsupported arguments, and required justification for the arguments, for example, when they discussed the effect of parameter *a* on the increase or decrease of a function.

Constructive use of authoritative information

The students used the teachers' (in our case, pre-service teachers) information constructively, planning and carrying out the activities by themselves. At the same time, they questioned the information provided by other students and demanded that activities be repeated in order to verify the findings. This may be explained by two factors: performing authentic activities and working with mobile phones. Simons (2004) maintained that authentic learning provides an environment that motivates the learners, involving them more deeply with their learning and placing them in control of it. Being in control of one's learning encourages constructive and critical use of authoritative information and of information provided by others. Mobile phones empowered the students and placed them in control of their learning, enabling them to enter the values obtained in real life contexts and turn these values into mathematical objects and relations. This may have helped the students feel in control of their learning from the first to the last phase. Mobile phones made authentic learning possible because they allowed the investigation of real life phenomena in the field (as opposed to investigating them retrospectively, after they returned to the classroom). The mobile phones also allowed learning which satisfies most of the characteristics of authentic learning described by Herrington et al. (2008). For example they allowed, as described above, social construction of knowledge, gave opportunities for reflection on the activities outcomes, enabled students' tacit knowledge of mathematics to be made explicit and discussed, encouraged the coaching and scaffolding by the preservice teachers only at critical times. This agrees with Lombardi (2007) who argues that the emergence of a new set of technological tools can offer students a more authentic learning experience based on experimentation and action.

Understanding as an emergent phenomenon

Brown et al. (1989) noted that Schoenfeld's teaching of problem solving (1985, 1991) deliberately attempts to generate mathematical practice and show college students how to think mathematically about the world, how to see the world through mathematicians' eyes, and thereby learn to use the mathematician's tools. In the second part of the experiment, students themselves suggested authentic activities in a real world setting to model with the help of a mobile phone. This enabled them to work as mathematicians, looking for real world phenomena to investigate and model mathematically. Thus, they learned, through relatively not complicated but rich mathematical actions, how to perform more complicated actions: how to think mathematically about the world, explaining logically the mathematical relations that modeled real-world phenomena.

Conclusions

Using mobile phones to teach and learn mathematics is still in its infancy. How would middle school students using these devices build their mathematical knowledge? I used new knowledge building models to verify this issue. The results imply that middle school students participating in the research project described in this paper completed all the knowledge building phases suggested by Gunawardena, Lowe and Anderson (1997). They also went through other knowledge building phases characteristic of the authentic context in which they learned.

What characterised the middle school students' knowledge building in the mobile phone environment is that they advanced their knowledge of ideas as a community, collaborating to engage in authentic activities with the help of mobile phones. They made constructive and critical use in general of information supplied by others, and in particular of authoritative information. In addition to these characteristics, students participating in the study worked as mathematicians, especially in the second part of the experiment, when they suggested real world phenomena to explore using the mobile phone.

The observations suggest that learning through authentic activities involving the use of mobile phones can encourage and enrich K12 students' knowledge building in mathematics.

Acknowledgment

Special thanks are due to the three preservice teachers who collected the data analysed in this research.

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